



Unifying boiling and degassing theories: Self-similar and pseudo-steady state analysis

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ABSTRACT

The two phenomena of degassing and boiling, regarding bubble growth in a liquid with a dissolved gas, are studied separately in literature with the corresponding theories to diverge from each other at the boiling temperature. Here a unifying approach is developed in terms of heat and mass transfer conservation laws to describe both phenomena. It is shown that the corresponding system of partial differential equations admits a self-similar solution for constant bubble growth rate conditions. The self-similarity solution is further simplified in order to indicate several asymptotic bubble growth regimes. The case of non-constant bubble growth rate conditions is treated only for the asymptotic regime of quasi-steady growth. Several results for idealized and realistic bubble growth cases are presented. It is shown that the two individual theories of degassing and boiling are unified through the appropriate manipulation of the bubble temperature. Small temperature differences can yield large variation in bubble growth rate in a small range around the boiling temperature. The system water-nitrogen is examined in detail and it is found to be outside of the domain of asymptotic theories (i.e. the complete self-similar solution has to be employed).

1. Introduction

Gas bubble formation and growth in a liquid supersaturated with dissolved gas is frequently referred as degassing [1]. This phenomenon is of particular importance to a number of technological applications. Some of these are the glass refining processes, polymeric foams production [2], beverages and wines production [3], bubble-based separation technologies (e.g. dissolved air flotation) [4]. Degassing is also important in human physiology (bubble creation in blood during decompression of divers [5]) and volcanic eruptions (bubble creation in magma) [6]. The significance of the process explains the large effort made in literature for the mathematical modeling and analysis of the problem. There are two modes of degassing, the decompression and the thermal one. Decompression degassing typically occurs at low temperature where a liquid can be assumed as non-volatile. Several studies for decompression degassing of single component and multicomponent gases in liquids of different rheology have been performed [7]. In the case of thermal degassing the temperature is usually raised and the liquid volatility should be taken into account since vapor appears as a considerable bubble component [8].

Degassing is a distinctly different bubble growth mechanism than

boiling. The driving force for bubble appearance in degassing is the supersaturation of dissolved gas (difference between concentration of dissolved gas and solubility). The driving force for bubble appearance in boiling is the superheat (difference between fluid temperature and boiling temperature). The kinetics of degassing is associated with mass transfer of the dissolved gas from the bulk fluid to the bubble surface. On the other hand, the kinetics of boiling is associated with heat transfer towards the bubble in order to supply the energy required for evaporation. Mathematical modeling of bubble growth in boiling has been studied more extensively than bubble growth in degassing [9]. Boiling efforts focus mainly on surface phenomena (pool, flow boiling) in which (as in thermal degassing) bubbles emerge on a hot solid surface. However, there are also studies on bulk boiling (e.g. cavitation induced by decompression [10]).

According to the above discussion the mathematical models of the two phenomena are completely different. What is noteworthy is that the degassing model diverges to infinite bubble growth rate and the boiling model leads to zero bubble growth rate as the temperature approaches the boiling one from low and high temperature side, respectively. The question arises on how to model the growth of a bubble created by degassing when the temperature increases to overcome the boiling

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point. Also, what is the bubble growth rate in a supersaturated liquid exactly at its boiling point? How noncondensable but soluble gases [11] affect boiling? A unified theory of degassing and boiling combined heat and mass transfer is developed here to answer questions like those above. It is noted that the consideration of combined heat and mass transfer during bubble growth has been considered also in the pioneering work of Scriven [12] but it is irrelevant to the present problem. That work refers to a purely boiling problem according to the present terminology. The solvent consists of two liquids with different vapor pressures so the mass transfer determines the composition at the bubble surface. Summarizing, the problem of multicomponent gases (degassing) and the problem of multicomponent liquid (boiling) have been considered in the past but the gas-vapor combination through the whole temperature regime has not. A mainly analytical study of this combination is the focus of the present work.

The structure of the present work is the following: At first the bubble growth mathematical problem for arbitrary fluid temperature is formulated. Then, it is shown that a self-similarity solution of the model exists for constant fluid conditions and zero initial bubble size. A simplification of the mathematical model for pseudo-steady state growth follows. Finally, several results for idealized and for realistic systems are presented.

2. Problem formulation

The scope here is to model the problem of a bubble growing in a volatile liquid with gas dissolved in it, for an arbitrary temperature history of the liquid including temperatures larger than the liquid boiling point. Although a quite general model for all bubble growth stages after the nucleation event could be set-up, some terms are of great complexity and small significance so it is preferable (for saving space) to ignore them right from the beginning than eliminating them at latter stages. These terms are assumed negligible as explained below.

A first assumption is that the gas density is much smaller than the liquid density which restricts the analysis to pressure levels up to 10 atm (larger pressures can be considered but with an increasing error with increasing pressure). A second assumption is the omission of the surface tension effect. This effect is indeed negligible except for very small bubbles (of the order of tenths of microns) [13] and so it is typically ignored in both degassing and boiling studies. A third assumption is the ignorance of the inertia regime of bubble growth. This regime appears at the first stages of growth and specifically in the boiling case where the bubble pressure exceeds the vapor pressure [9]. In any case, this regime is of importance for a limited parametric space and can be ignored without significant error.

The solution of the Navier-Stokes equation in the liquid around the growing bubble leads to the so called Rayleigh-Plesset equation which relates the pressure in the bubble with liquid characteristics [14]. It can be shown that under typical conditions most of the terms in this equation can be eliminated yielding a trivial expression for the pressure in the bubble

$$P_b = P_{ex} = P_g + P_v \quad (1)$$

where the bubble pressure P_b is equal to the external (liquid) pressure P_{ex} and P_v , P_g are the vapor pressure and gas pressure in the bubble, respectively. The vapor pressure is a function of temperature and can be found tabulated. A nice analysis for a possible exception from the above case, requiring the inclusion of a viscous damping term in equation (1), for a specific volcanic system is reported in Ref. [15].

The solution of continuity equation in the liquid leads to the following spherically symmetric velocity profile $u_r = \dot{R}R^2/r^2$ where R is the bubble radius, r is the radial coordinate and $\dot{R} = dR/dt$. The conservation equations for the dissolved gas and for the heat include accumulation, convection and diffusion/conduction terms and can be written in spherical coordinates as

$$\frac{\partial c}{\partial t} + \frac{R^2 \dot{R}}{r^2} \frac{\partial c}{\partial r} = \frac{D}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial c}{\partial r} \right) \quad (2)$$

$$\frac{\partial T}{\partial t} + \frac{R^2 \dot{R}}{r^2} \frac{\partial T}{\partial r} = \frac{\alpha}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) \quad (3)$$

where $c(r,t)$ is the dissolved gas concentration, $T(r,t)$ is the liquid temperature, t is the time, D is the gas diffusivity in the liquid and α is the thermal diffusivity of the liquid. The gas-in-gas diffusivity inside the bubble is orders of magnitude larger than the gas-in-liquid diffusivity outside the bubble therefore the internal bubble composition can be assumed uniform through its volume. More specifically, the vapor molar fraction is $x = P_v/P_{ex}$ and the gas molar fraction is $1-x$.

The heat capacity per unit of volume of the gas is order of magnitude smaller than that of the liquid so the thermal inertia of the bubble can be ignored leading to a pseudo-equilibrium between the heat entering the bubble by conduction and the heat required for phase change of the liquid. In addition, the gas solubility in the liquid is small so its effect on vapor pressure can be ignored. Based on the above assumptions, a balance for the gas entering the bubble leads to the relation (see Appendix A for details):

$$SD \left(\frac{\partial c}{\partial r} \right)_{r=R} = \frac{dV \rho_g}{dt} \quad (4)$$

where $S = 4\pi R^2$ is the surface area of the bubble and $V = 4\pi R^3/3$ is the bubble volume. The gas density ρ_g is given from the ideal gas law as $\rho_g = P_g/R_g T$ where R_g is the gas constant. It is noted here that form equation (1) $P_g = P_{ex} - P_v$.

The balance for the heat entering the bubble is written as (see Appendix A for details)

$$Sk \left(\frac{\partial T}{\partial r} \right)_{r=R} = \Delta H \frac{dV \rho_v}{dt} \quad (5)$$

where k is the liquid conductivity, $\rho_v = P_v/RT_b$ and ΔH is the latent heat computed at the temperature of the bubble T_b which is equal to the liquid temperature at $r = R$. The heat of gas dissolution was shown to be insignificant in Ref. [16]. The closure of the above mathematical problem is achieved by considering the thermodynamic relation $P_v = P_v(T_b)$. The essential suggestion in the present work is that both equations (4) and (5) should be fulfilled in any case and not only the one of these as is the case for degassing and boiling.

The boundary condition for the gas concentration field at $r = R$ has the form $c = HP_g$

where H is the so called Henry constant computed at the bubble temperature. The far field conditions (regarding the bubble scale) are $c = c_f$ and $T = T_f$ at $r \rightarrow \infty$. The initial condition is that at $t = 0$ the bubble size has a given value. The system of equations (1)–(5) consists a well defined mathematical problem which describes the evolution of bubble size, composition and temperature for any imposed evolution of the external forcing functions (fluid conditions), namely $c_f(t)$, $T_f(t)$ and $P_{ex}(t)$.

3. Self-similar treatment

The numerical treatment of the above system of equations for arbitrary external forcing functions is complex and does not allow a deep insight to the physics of the problem. In order to understand better the effect of various parameters on bubble growth, some specific cases are examined next that afford analytical treatment of the mathematical problem. The simplest case regarding the three external forcing functions corresponds to three fixed single values (with no time variation). Let us denote these values as c_f , T_f , P_f . The condition for having bubble growth is the gas solubility under the conditions in the bubble c_e to be smaller than c_f , thus triggering solute mass transfer towards the bubble.

We will seek a self-similar solution for this problem. This implies a solution with constant temperature and composition in the bubble with respect to time and the respective liquid phase profiles being functions of only the new variable $\eta = r/R(t)$. In this case the fixed in time bubble temperature T_e , and gas solubility c_e , can be used to introduce normalized variables as follows:

$$C = \frac{c - c_f}{c_e - c_f}, \quad \tau = \frac{T - T_f}{T_e - T_f} \quad (6a,b)$$

Using these new variables equations (2) and (3) remain the same with only change the replacement of c and T by C and τ , respectively. Nevertheless, the other model equations are changed significantly. In particular equations (4) and (5) take the form

$$\frac{dR}{dt} = DF \left(\frac{\partial C}{\partial r} \right)_{r=R} \quad (7)$$

$$\frac{dR}{dt} = g \left(\frac{\partial \tau}{\partial r} \right)_{r=R} \quad (8)$$

where

$$F = \frac{(c_f - c_e)R_g T_e}{P_g}, \quad g = \frac{(T_f - T_e)R_g T_e k}{P_v \Delta H} \quad (8a)$$

The system of governing equations must be solved with boundary conditions $C = 1, \tau = 1$ at $r = R$ and $C = 0, \tau = 0$ at $r \rightarrow \infty$. The initial conditions are that $C = 0, \tau = 0$ everywhere and that the initial bubble size is $R = R_0$. The substitution of the required dependencies $C(r,t) = C(\eta)$ and $\tau(r,t) = \tau(\eta)$ in the governing equations (modified eqs (2) and (3)) leads after some algebra to

$$\frac{d^2 C}{d\eta^2} + \left[\frac{2}{\eta} + \frac{R\dot{R}}{D} \left(\eta - \frac{1}{\eta^2} \right) \right] \frac{dC}{d\eta} = 0 \quad (9)$$

$$\frac{d^2 \tau}{d\eta^2} + \left[\frac{2}{\eta} + \frac{R\dot{R}}{\alpha} \left(\eta - \frac{1}{\eta^2} \right) \right] \frac{d\tau}{d\eta} = 0 \quad (10)$$

The requirement for the problem to depend only on η is fulfilled only if $R\dot{R}$ is a constant value, let us denote it β , and $R_0 = 0$. The second requirement is not really important because as the ratio R/R_0 increases with time the actual solution converges to the self-similar one. In particular, in case of problems including nucleation, the nucleus size is typically very small compared to bubble sizes of interest so the self-similarity solution is accurate. Substituting β in equations (9) and (10) and subsequently integrating with respect to η from 1 to infinity using the corresponding boundary conditions leads to

$$C = \int_{\eta}^{\infty} \frac{1}{x^2} \exp\left(-\frac{\beta x^2}{2} - \frac{\beta}{x}\right) dx \left[\int_1^{\infty} \frac{1}{x^2} \exp\left(-\frac{\beta x^2}{2} - \frac{\beta}{x}\right) dx \right]^{-1} \quad (11)$$

$$\tau = \int_{\eta}^{\infty} \frac{1}{x^2} \exp\left(-\frac{D\beta x^2}{2\alpha} - \frac{D\beta}{\alpha x}\right) dx \left[\int_1^{\infty} \frac{1}{x^2} \exp\left(-\frac{D\beta x^2}{2\alpha} - \frac{D\beta}{\alpha x}\right) dx \right]^{-1} \quad (12)$$

Substitution of (11) and (12) in (7) and (8) leads after some algebra to the following final result for the bubble size evolution:

$$R = \sqrt{2\beta Dt} \quad (13)$$

where

$$\beta = \frac{F e^{-3\beta/2}}{S(\beta)} \quad (14)$$

$$\beta \lambda = \frac{G e^{-3\beta\lambda/2}}{S(\beta\lambda)} \quad (15)$$

$$\text{where } S(y) = \int_1^{\infty} \frac{1}{x^2} \exp\left(-\frac{yx^2}{2} - \frac{y}{x}\right) dx \quad (16)$$

The parameter $\lambda = D/\alpha$ is a physical parameter of the liquid related to the relative values of mass to heat transport dynamics. This parameter can be also considered as the ratio of Prantl to Schmidt number. The dimensionless parameter G is given as

$$G = \frac{\rho_l c_{pl} (T_f - T_e) R_g T_e}{P_v \Delta H} \quad (17)$$

In summary, the bubble grows in proportionality to $(Dt)^{0.5}$. The proportionality coefficient β should obey the conditions (14) and (15) in which the dimensionless constants λ (already discussed), F and G appears. The parameter F becomes the only parameter in case of ignoring the heat transfer effect on degassing which is the typical case examined in literature. In many studies this parameter has not a specific name but in others the name ‘‘Foaming number’’ is used inspired by polymer applications [17]. This number is the ratio of the residual (above solubility) gas volume to the liquid volume in which it is dissolved. For instance, a foaming number equal to 10 means that the excess dissolved gas in the liquid, has volume ten times larger than the volume of the liquid. The parameter G appears in pure boiling studies and it is the amount of superheat of a liquid volume to the amount of latent heat of the same volume of vapor. The coupling between these two parameters through the vapor pressure and bubble temperature appears for first time in the present work.

Equation (14) has two well known asymptotic solutions in the limits of small and large F . In case of $F \ll 1$ its solution is $\beta = F$ and in case of $F \gg 1$ its solution is $\beta = \frac{6}{\pi} F^2$. It is stressed that the first order correction for small F expansion is of order $F^{1/2}$ so F must be really small for the asymptotic result to hold. From the physical point of view, the small F limit corresponds to pseudo-steady state conditions (diffusion dominated) whereas the large F limit to convection domination with a concentration boundary layer much smaller than the bubble radius (permitting ignorance of the interface curvature). Equation (15) can be expanded in the limits of small and large G in a similar way with (14). The combined consideration of asymptotes for equations (14) and (15) leads to a total of four asymptotic cases:

Case I $F \ll 1, G \ll 1$

This case corresponds to pseudo-steady state conditions (e.g. Refs. [18,19]) (steady state diffusion and conduction equation). This case mainly refers to low temperature conditions and leads to

$$\beta = F \quad (18a)$$

$$G = \lambda F \quad (18b)$$

Case II ($F \gg 1, G \ll 1$)

This case corresponds to convection dominated mass transfer but conduction dominated heat transfer. This is a very probable situation since mass diffusivity is much smaller than thermal diffusivity so there is a range of bubble growth velocities for which this condition hold. The asymptotic result is

$$\beta = \frac{6}{\pi} F^2 \quad (19a)$$

$$G = \frac{6}{\pi} \lambda F^2 \quad (19b)$$

Case III ($F \gg 1, G \gg 1$)

In this case both mass and heat transfer are convection dominated.

This case is very probable to occur for liquid temperature much larger than the regular boiling temperature. The result is

$$\beta = \frac{6}{\pi} F^2 \quad (20a)$$

$$G = \lambda^{1/2} F \quad (20b)$$

Case IV ($F \ll 1$, $G \gg 1$)

In this case mass transfer is diffusion dominated and heat transfer is convection dominated. This case is quite improbable in practice. Equation (21b) implies that $\lambda \gg 1$ which is not the case for the known gas-liquid systems. The result is

$$\beta = F \quad (21a)$$

$$G = \left(\frac{\pi}{6} \lambda F\right)^{1/2} \quad (21b)$$

From a mathematical point of view, the difference between the system of equations (14) and (15) and the asymptotic analysis equations is that (14) and (15) must be solved simultaneously whereas the asymptotic analysis leads to a direct relation of growth factor to the physical parameters of the problem (i.e. equations (18a), (19a) and (20a), (21a) and to a compatibility condition (i.e. (18b), (19b), (20b), (21b)) which can be used for the bubble temperature determination. It is interesting that the domination of molecular (diffusion/conduction) or convective transport is determined without any association to the actual bubble growth velocity. This is due to the fact the both molecular and convection contribution to growth depends on the same way on bubble size so their ratio is a constant depending on physical parameters and not on bubble size or time. The requirements on F and G are not enough to ensure molecular transport or convection domination on growth. The values of λ must be such that allow simultaneously the fulfillment of these restrictions and of the compatibility condition.

4. A special case for variable external conditions

The problem of bubble growth for arbitrary but constant external conditions has been analyzed in the previous section. Yet, in practice the external conditions can be time dependent. A special case in which the complete mathematical problem of equations (1-5) can be considerably simplified is the one of small F and small G (which for example corresponds to a system with small gas solubility and large latent heat of liquid evaporation). Under this conditions equations (2) and (3) degenerate to the steady state diffusion and conduction equations which can be solved analytically to give the following concentration and temperature profiles.

$$c = c_f(t) + \frac{(HP_g - c_f(t))R}{r} \quad (22)$$

$$T = T_f(t) + \frac{(T_b(t) - T_f(t))R}{r} \quad (23)$$

where $T_b(t)$ is the instantaneous bubble temperature. The mass and temperature fluxes computed from the above profiles are substituted to the total balances in equations (4) and (5) to give:

$$\frac{d}{dt} \left(\frac{VP_g}{R_g T_b} \right) = 4\pi DR (c_f - HP_g) \quad (24)$$

$$\frac{d}{dt} \left(\frac{VP_v \Delta H}{R_g T_b} \right) = 4\pi kR (T_f - T_b) \quad (25)$$

The above system of differential equations is implicit with respect to the unknown functions R and T_b . An explicit system is constructed by

introducing the new variables $X = \frac{VP_g}{R_g T_b}$, $Y = \frac{VP_v}{R_g T_b}$. Assuming ΔH is temperature independent the governing system of equation takes the form

$$\frac{dX}{dt} = (4\pi)^{2/3} 3^{1/3} D (X + Y)^{1/3} \left(\frac{R_g T_b}{P_f} \right)^{1/3} (c_f - H(P_f - P_v)) \quad (26)$$

$$\frac{dY}{dt} = (4\pi)^{2/3} 3^{1/3} \frac{k}{\Delta H} (X + Y)^{1/3} \left(\frac{R_g T_b}{P_f} \right)^{1/3} (T_f - T_b) \quad (27)$$

where P_v and H are given functions of T_b with T_b given at each instant from the solution of the transcendental equation $P_v(T_b) = P_f/(1 + X/Y)$. The bubble radius and gas pressure are computed from X and Y as:

$$R = (X + Y)^{1/3} \left(\frac{3R_g T_b}{4\pi P_f} \right)^{1/3} \quad (28)$$

$$P_g = P_f - P_v$$

The vapor molar fraction in the bubble is $1/(1 + X/Y)$. For a specific initial bubble size and composition the above system can be integrated with the corresponding initial conditions for X and Y. In case of zero initial bubble size, the initial X, Y are zero and inspection of the right hand side of the equations implies that the bubble radius stays at zero. The solution to this problem is to consider very small initial equal values for X and Y. This implies a wrong initial composition value but the system very soon converges to the right value which in case of constant external variables is the self-similar one (as it should be). So the self-similar approach requires zero initial bubble size whereas the dynamic approach requires non-zero initial bubble size but both approaches converge to each other as the bubble grows.

5. Results and discussion

Indicative results for some simplified and some realistic cases are shown in this section. The simplified cases are analyzed in order to understand better the structure of the unified degassing-boiling problem and the influence of physical parameters on bubble growth.

i) quasi-steady bubble growth under fixed external conditions

Let us consider the problem of bubble growth from zero size under fixed external conditions P_f , T_b , c_f . This problem has a self-similar solution as it was shown in the previous section. In case of very small F and G numbers (growth under pseudo-steady conditions-Case I of section 3) the governing equations are:

Bubble evolution equation (based on equation (18a)):

$$R^2 = 2D \frac{(c_f - c_b) R_g T_b}{P_f - P_v} t \quad (29)$$

Compatibility equation (based on equation (18b)):

$$\frac{k(T_f - T_b)}{P_v \Delta H} = \frac{D(c_f - c_b)}{P_f - P_v} \quad (30)$$

where c_b is the gas solubility at bubble temperature T_b . A further simplification is to consider the large supersaturation limit $c_f \gg c_b$ introduced in Ref. [15]. From the physical point of view this limit can be achieved by dissolving the gas in a pressure much larger than the one prevailing during bubble growth. It is stressed that the focus of the present analysis is the degassing-boiling transition which means that the temperature is close to the regular boiling point and that the gas solubility is very small (reduced Henry coefficient and reduced partial gas pressure due to increased vapor pressure). So the large supersaturation limit fits perfectly to the conditions of interest here. Equation (30) takes the form

$$\left(\frac{P_f}{P_v} - 1\right)(T_f - T_b) = \frac{Dc_f\Delta H}{k} \quad (31)$$

Equation (31) must be solved to give the bubble temperature T_b but before proceeding with it, the vapor pressure – temperature relation must be specified. The simplest possible (but still qualitatively correct) vapor pressure curve will be considered here: the vapor pressure is zero up to a temperature T_A and then it increases linearly with temperature i. e. $P_v = 0$ for $T_b < T_A$ and $P_v = \mu(T - T_A)$ for $T_b \geq T_A$. Let us call T_B the regular boiling temperature at the pressure of the experiment P_f . Finding T_B from the boiling condition $P_f = P_v$ leads after some algebra to ($T_A < T_b < T_B$):

$$\frac{P_v}{P_f} = \frac{T_b - T_A}{T_B - T_A} \quad (32)$$

Introducing the temperature normalization $\bar{T} = \frac{T - T_A}{T_B - T_A}$ equations (31) and (32) are combined to give:

$$\left(\frac{1}{\bar{T}_b} - 1\right)(\bar{T}_f - \bar{T}_b) = \frac{Dc_f\Delta H}{k(T_B - T_A)} \quad (33)$$

The right hand side of the above equation defines a new dimensionless parameter which is denoted as Z . This parameter has a very specific meaning: it is the ratio of the heat needed for bubble growth to the heat given to the bubble by conduction from the liquid. The parameter has not a specific scale thus absolute values does not mean anything. Nevertheless as Z increases the limitations to bubble growth imposed by heat transfer dynamics, increase.

Equation (33) is solved for the bubble temperature to give

$$\bar{T}_b = \frac{1}{2} \left(1 + \bar{T}_f + Z - \left((1 + \bar{T}_f + Z)^2 - 4\bar{T}_f \right)^{0.5} \right) \quad (34)$$

With the bubble temperature at hand, the next step is to compute the bubble evolution curve. Unfortunately, this depends on the absolute temperature in Kelvin degrees (through the ideal law for gas density). A reasonable approximation is to assume that $(T_B - T_A)/T_B \ll 1$ so $T_b \approx T_f$ for the gas density estimation (but not for the vapor pressure computation). This leads to

$$R = \left(2D \frac{c_f R_g T_b}{P_f} B t \right)^{0.5} \quad (35)$$

where the parameter B is given as $1/(1 - \bar{T}_b)$. The bubble growth problem under the examined conditions has been completely solved through equations (34) and (35). Let us give some explicit result for the growing bubble as a function of the external temperature when it approaches the regular boiling point from either higher or lower temperature values. The bubble temperature dependence on external temperature is shown in Fig. 1. The actual curve interpolates two asymptotes: The low temperature asymptote corresponding to degassing ($\bar{T}_b = \bar{T}_f$) and the large temperature asymptote corresponding to boiling ($\bar{T}_b = 1$). The width of the transition zone depends on the value of Z . The higher the value of Z the more severe the heat transfer limitations and the higher the difference between external temperature and bubble temperature. In case of the smallest value of Z used here ($Z = 0.001$) the thickness of the transition zone appears negligible in the figure and the result looks similar to the one of considering separately the degassing and boiling phenomena for $\bar{T}_f \leq 1$ and $\bar{T}_f \geq 1$, respectively.

The temperature dependent growth parameter B is shown as function of the external temperature in Fig. 2. The low temperature (degassing) asymptote corresponds to B proportional to $P_f/(P_f - P_v)$ and the large temperature (boiling) asymptote corresponds to B proportional to $T_f - T_B$. It is stressed that the present model predicts a smooth growth rate increase as the temperature overcomes the regular boiling point. Whereas the $Z = 0.001$ case in Fig. 1 was shown to correspond to separated growth mechanisms in terms of bubble temperature, the situation is

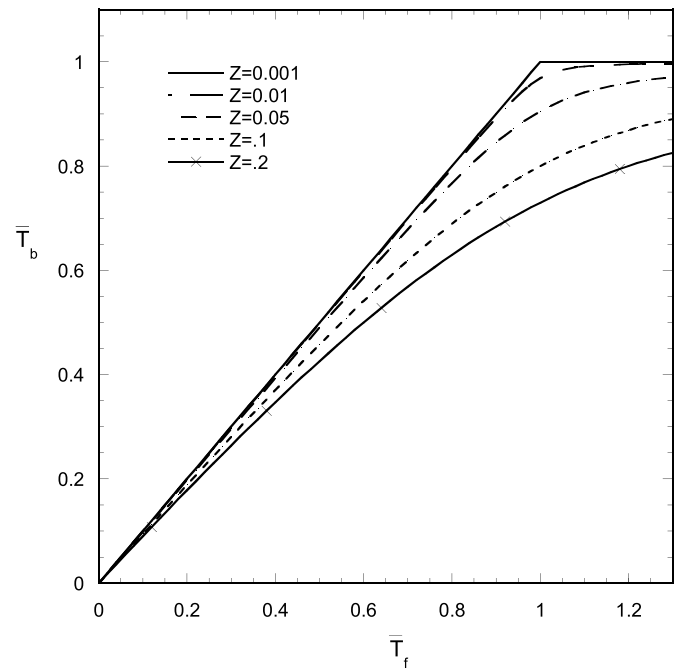


Fig. 1. Normalized bubble temperature versus normalized external temperature for several values of parameter Z (for $F, G \ll 1$).

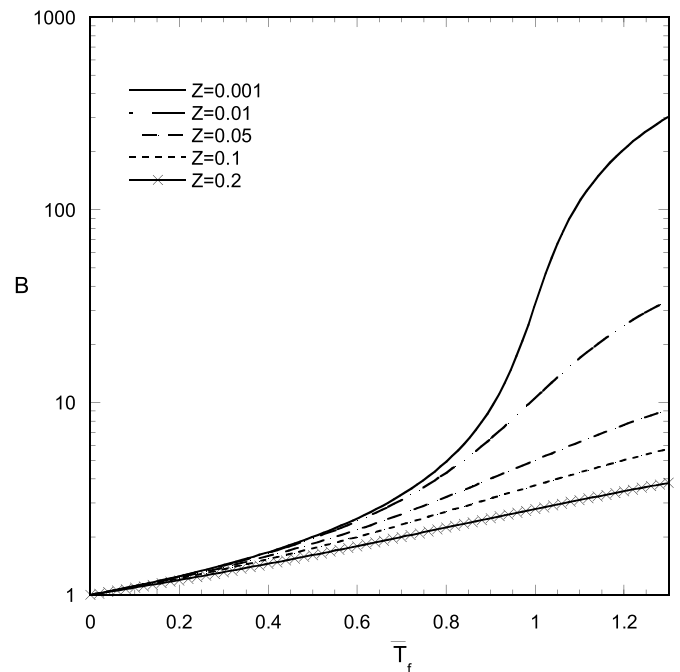


Fig. 2. Bubble growth factor B versus normalized external temperature for several values of parameter Z (for $F, G \ll 1$).

quite different regarding bubble growth rate in Fig. 2. The separated growth mechanisms predicts that B goes to infinity as T_f approaches the boiling point from lower temperatures (degassing) whereas it goes to zero as T_f approaches boiling point from higher temperatures. The present model predicts slight temperature differences between T_b and the two asymptotes (being too small to appear in Fig. 1) which lead to huge differences for B allowing the diverged behavior of the separated mechanisms to be replaced by a smooth curve (see Graphical Abstract for a qualitative point of view). The unified model is primarily scaled with respect to degassing and the factor B could be considered as

degassing rate enhancement due to liquid evaporation. It is worth noticing that even in the case of degassing (temperature smaller than the boiling point) the dominant growth mechanism could still be heat transfer through conduction.

ii) quasi-steady bubble growth under variable external temperature

In many practical cases the external conditions vary during bubble growth. For example the bubble can be created and start its growth in the degassing regime but the growth can be continued in the boiling regime. A typical example is in potato frying where water mass loss starts before the potato temperature reach the boiling point [20]. An alternative way to cross the two regimes is by lowering the pressure instead of increasing the temperature. The bubble growth process itself can lead to variation of the external temperature and concentration through the consumption of heat and dissolved gas mass. The simplest case of bubble growth under variable external conditions is the pseudo-steady state corresponding to small F and G values. The starting point for the analysis is equations (26) and (27). Initially the assumptions of large supersaturation, and of an ideal linear vapor pressure – temperature dependence demonstrated in the previous example, are introduced. In addition, it is assumed that the simulations refer to a narrow range of temperature (e.g. up to 20 K degrees) so the gas density variation in the bubble can be neglected. In that case the density can be computed at the regular boiling point T_B .

The governing equations take the form (the temperature normalization is the same with the one of the previous example):

$$\frac{dR^3(1 - \bar{T}_b)}{dt} = \frac{3DR_g T_B c_f}{P_f} R \quad (36)$$

$$\frac{dR^3 \bar{T}_b}{dt} = \frac{3kR_g T_B (T_B - T_A)}{P_f \Delta H} R (\bar{T}_f - \bar{T}_b) \quad (37)$$

A reference bubble radius R_0 is introduced and the actual radius is normalized with respect to this (i.e. $R_a = R/R_0$). A variable transformation similar to the one shown in equations (26) and (27) leads to:

$$\frac{dX}{dt_a} = (X + Y)^{1/3} \quad (38)$$

$$\frac{dY}{dt_a} = \frac{1}{Z} \left(\bar{T}_f - \frac{Y}{X + Y} \right) (X + Y)^{1/3} \quad (39)$$

where the dimensionless time t_a is defined as

$$t_a = \frac{3R_0^2 DR_g T_B c_f}{P_f} t \quad (40)$$

$$\text{and } R_a = (X + Y)^{1/3} \text{ and } \bar{T}_b = Y/(X + Y) \quad (41)$$

The above system degenerates to equations (34) and (35) in case of constant temperature T_f . In case of variable T_f the integration is performed numerically starting from very small values of X and Y in order to avoid discontinuities. A linear increase of external temperature is considered next (i.e. $\bar{T}_f = 0.2t_a$). The bubble is assumed to nucleate at the temperature T_A of vapor pressure appearance and grows as the external temperature increases. The evolution in time of the normalized bubble temperature is shown in Fig. 3. As in the case of constant external temperature, the $Z=0$ case corresponds to separate phenomena of degassing ($\bar{T}_b = \bar{T}_f$ for $\bar{T}_f < 1$ i.e. for $t_a < 5$) and boiling ($\bar{T}_b = 1$ for $\bar{T}_f \geq 1$ i.e. for $t_a \geq 5$). It is noted that for the same value of Z the bubble temperature deviates more from the two asymptotes than in the case of constant external conditions examined above. This due to the fact that bubble growth starts from low temperature with large gas fraction. This gas quantity remains in the bubble as temperature increases lowering the bubble temperature in the boiling regime. The bubble size evolution corresponding to the temperature results of Fig. 3 is shown in Fig. 4. The

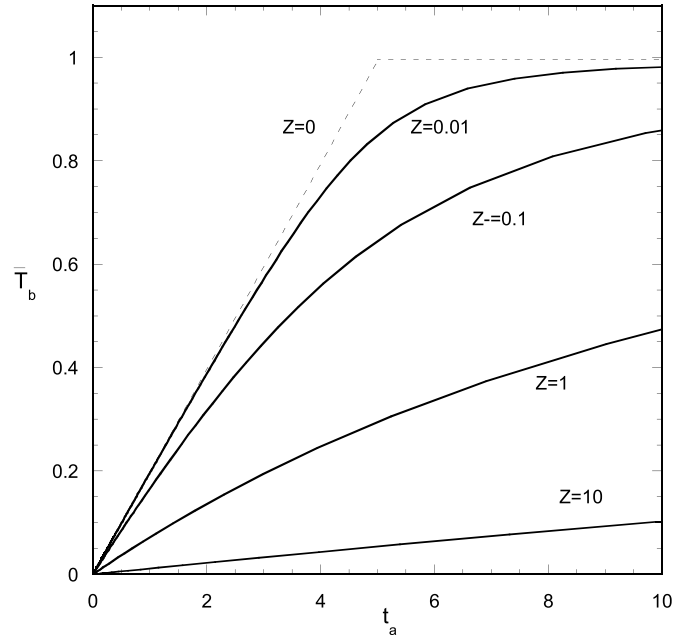


Fig. 3. Normalized bubble temperature evolution for linearly increasing in time external temperature (for $F, G \ll 1$).

growth curve do not correspond to the well known square root with time behavior since not only the driving force but also the growth regime changes from degassing to boiling as the bubble grows.

A more complex external temperature pattern is considered next. The external temperature oscillates between a minimum temperature $T_f = T_A$ ($\bar{T}_f = 0$) and a maximum temperature $\bar{T}_f = 2$, deep in the boiling regime. In this case, as the bubble grows there are multiple transitions from degassing to boiling regime and reversely. The bubble temperature and radius profiles for two values of Z are shown in Fig. 5 together with the external temperature profile. The system response does not look periodic like the external disturbance since it takes of few

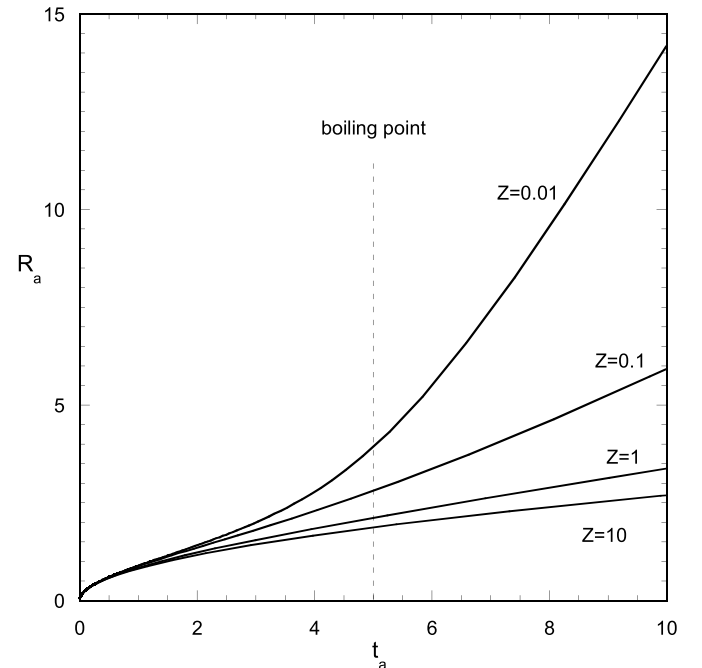


Fig. 4. Normalized bubble radius evolution for linearly increasing in time external temperature (for $F, G \ll 1$).

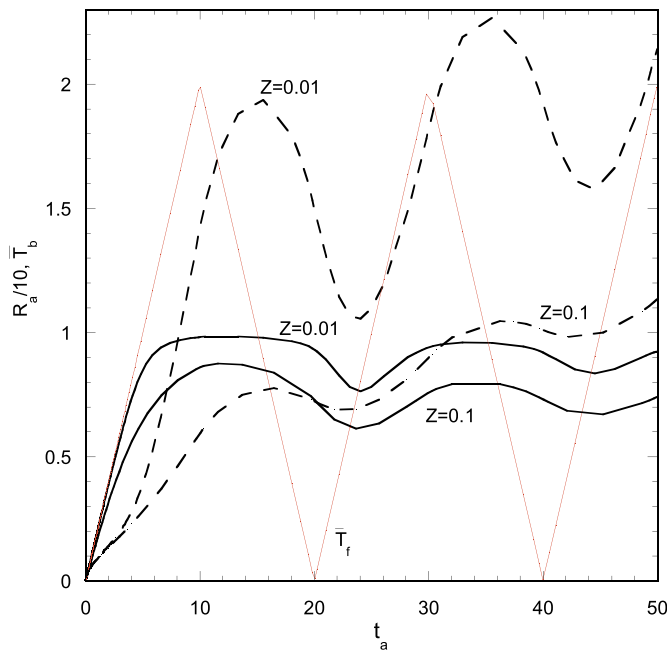


Fig. 5. Normalized bubble temperature (continuous lines) and bubble radius (dashed lines) evolution for a periodically varying external temperature (shown also in the figure) and for two values of parameter Z .

cycles before establishing periodicity. The bubble temperature profile is of particular interest since it appears to overdamp the external temperature disturbance. The bubble temperature does not follow the external temperature not only at its increase over the boiling temperature but also at its decrease. The non-monotonic evolution of bubble radius is due to vapor recondensation.

iii) Bubble growth in a physical system (nitrogen in water)

After the analysis performed for simplified systems, results for a realistic problem are now presented. The physical problem is the following: Water is saturated with respect to nitrogen at pressure P_d (larger than atmospheric) and temperature 20°C . Then the system undergoes decompression to $P_f = 1\text{ atm}$ with simultaneous raise of temperature to the value T_f since we want to study the combined effect of temperature and of nitrogen concentration. The above procedure leads to supersaturation with respect to dissolved nitrogen and thus to bubble nucleation and growth. Then the question is how the bubble size evolves with respect to dissolution pressure P_d and temperature T_f . Based on the analysis in section 3, the problem admits a self-similarity solution but no other simplification is possible since the parameters F and G are determined from the physics of the problem and as such can take any values. The solution of the self-similarity problem is given by equations (13–16). The parameters F and G are given in this case as

$$F = \frac{(c_f - c_b)R_g T_b}{P_f - P_v}, \quad G = \frac{\rho_l c_{pl}(T_f - T_b)R_g T_b}{P_v \Delta H} \quad (42)$$

As shown in Fig. 6 the temperature difference between bubble (T_b) and bulk liquid ($T = T_f$) is smaller than 1 K so the liquid physical properties can be assumed spatially uniform and can be computed at the liquid temperature T_f . It is noted that self-similar solutions at limiting cases can be derived even for spatially dependent properties (e.g. Refs. [21,22]). In any case, it is crucial to incorporate property temperature dependence because their variation with T_f can be large (especially true for diffusivity D). The water density, specific heat capacity, latent heat and thermal diffusivity are taken from literature tables. The vapor pressure of water is computed using Antoine law, the nitrogen in water

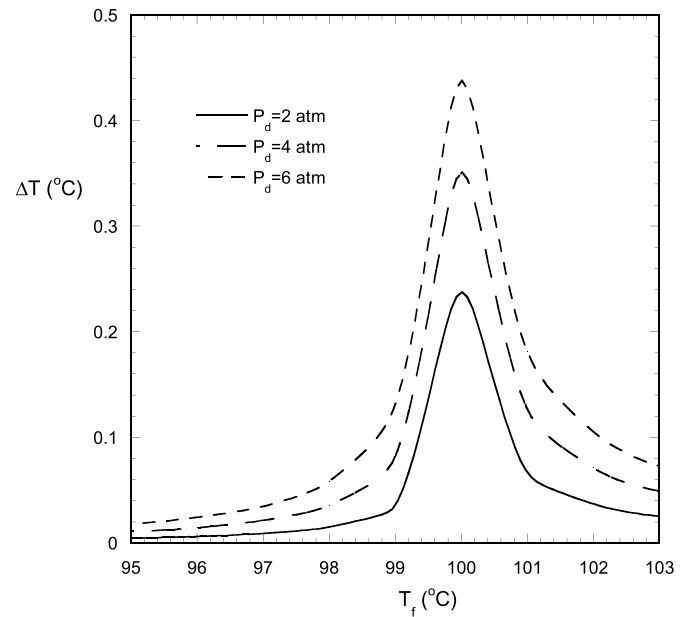


Fig. 6. Temperature difference $\Delta T = T_f - T_b$ between liquid and bubble versus external temperature for the nitrogen-water system and several values of dissolution pressure.

diffusivity is taken by interpolating experimental data from literature [23] and the concentrations c_f , c_b are computed from Henry law as: $c_f = H(T = 20^\circ\text{C})P_d$, $c_b = H(T_b)P_f$.

The problem is to find the pair of values β , T_b which fulfills the system of equations (14–16). The integral (16) is transformed to a differential equation and is computed numerically using the appropriate solver [24]. A Newton-Raphson procedure is developed to find the two unknowns of the system through an iterative procedure. The computations are performed for three values of the dissolution pressure P_d and for several values of the temperature T_f . Results are shown only for a narrow range of temperature around the boiling point of 100°C since outside this zone the new combined model approaches the approximate models of degassing or boiling.

The temperature difference between bubble and fluid/boiling temperatures (i.e. $\Delta T = T_f - T_b$ for $T_f < 100^\circ\text{C}$ and $\Delta T = 100^\circ\text{C} - T_b$ for $T_f \geq 100^\circ\text{C}$) is presented in Fig. 6. It is reminded that for the individual models for degassing and boiling this difference is zero leading to inconsistency of the growth rate between two regimes. The peak in ΔT is at the regular boiling point in which the deviation between the combined model and the individual ones is maximum as it has also been shown in previous examples. The larger P_d leads successively to higher nitrogen concentration, to higher mass transfer rate, to higher growth rate, to higher evaporation rate and to larger temperature reduction due to the heat requirements of evaporation. The absolute values of ΔT appear to be small (less than 0.5 K) but they induce large corrections to the bubble growth rate and allow the unification of the two independent bubble growth theories. For the particular physical problem examined the unified model must be necessary used for external temperatures between 95 and 105°C whereas the individual models are accurate outside of this region. The corresponding β values are shown in Fig. 7. The effect of P_d decreases as the temperature increases which is expected since as the process gets inside the boiling regime the effect of dissolved gas diminishes. The value of the parameters F and G for the cases examined are shown in Figs. 8 and 9. This is important in order to assess the adequacy of resorting to asymptotic cases for the examined physical system. In the particular temperature zone for which results are shown, no asymptote can be used and this why the full self-similarity result is used. Further study shows that Case I asymptote can be relevant for small temperature and P_d , Case II for intermediate temperature (still

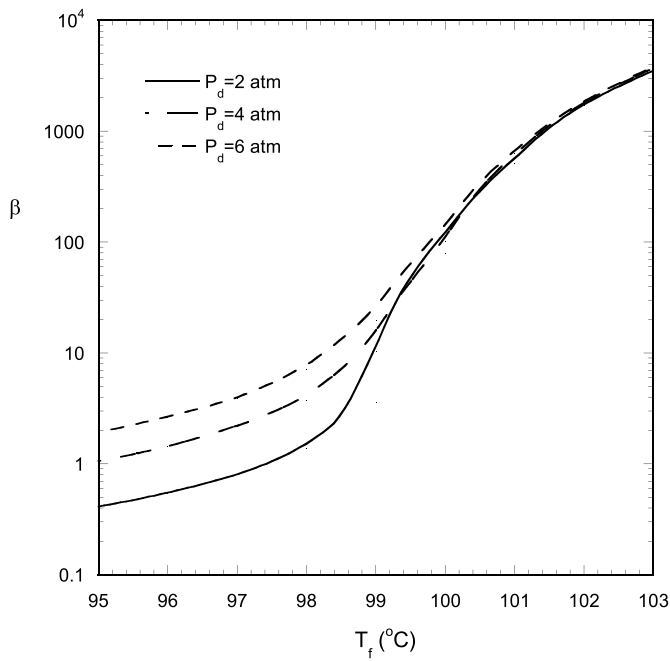


Fig. 7. Bubble growth factor β versus external temperature for the nitrogen-water system and several values of dissolution pressure.

smaller than 95 °C) and Case III for large temperature (larger than 103 °C).

In order to clarify the findings of the present work a short discussion follows. The answer to the question what is the growth rate of a bubble in natural water (saturated in air) at 100 °C is (i) infinite according to degassing theory and (ii) zero according to boiling theory. Both theories have been proven experimentally to be correct far from 100 °C. Here it is shown that they are correct everywhere except at a thin temperature window around 100 °C. At this window the new theory provides a finite bubble growth rate. It is achieved by adjusting the bubble temperature

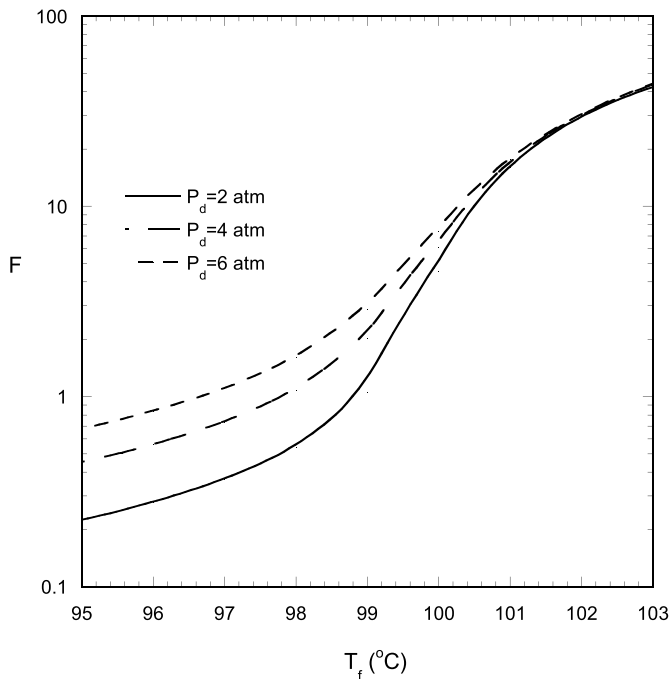


Fig. 8. Foaming number F versus external temperature for the nitrogen-water system and several values of dissolution pressure.

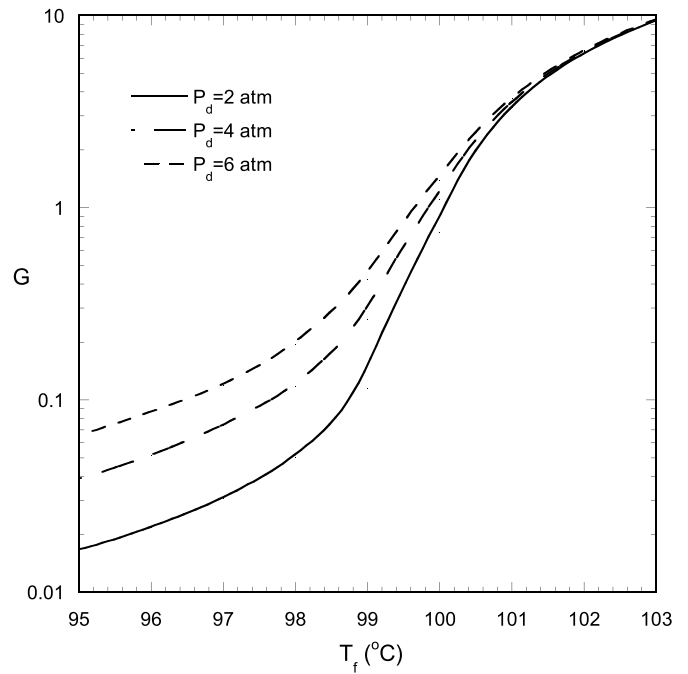


Fig. 9. Dimensionless parameter G versus external temperature for the nitrogen-water system and several values of dissolution pressure.

allowing a small difference with respect to the liquid temperature. This is already a huge improvement over the previous situation. A pictorial representation of the significance of the new approach with respect to the previous situation appears in Fig. 10.

6. Conclusions

The simultaneous consideration of heat and mass transfer during bubble growth in a volatile liquid which is supersaturated with dissolved gas allows the development of a generalized bubble growth model valid for any set of external conditions (temperature, concentration and pressure). It is shown that the mathematical model admits a self-similar solution for the case of constant external conditions and zero initial bubble size. The self-similar solution can be further simplified under specific set of restrictions for the physical parameters. The analysis of bubble growth for several ideal and realistic problems suggest that the new unified model differs from the individual models of degassing and boiling only in a narrow temperature zone around the boiling temperature. The new unified model connects the (divergent at the boiling

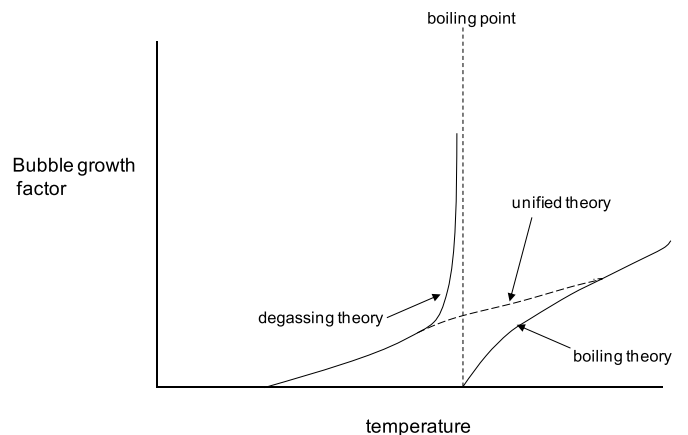


Fig. 10. A pictorial qualitative description of the significance of the new unified theory on bubble growth rate computation.

temperature) growth rates of the individual models by lowering the bubble temperature with respect to that of the external fluid.

Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Nomenclature

c	dissolved gas concentration
C	normalized concentration
c_{pb}	specific heat capacity in bubble
D	dissolved gas diffusivity
D_{gv}	binary gas-vapor diffusivity
F, g	parameters defined in equation (8a)
G	dimensionless parameter defined in equation (17)
H	Henry constant
k	thermal conductivity of liquid
k_b	thermal conductivity in bubble
K	evaporation rate
P_A, P_B, μ	parameters of the simplified vapor pressure-temperature dependence
P_b	bubble pressure
P_d	dissolution pressure
P_{ex}	liquid pressure
P_g	gas pressure
P_v	vapor pressure
R	bubble radius
r	radial coordinate
R_a	R/R_0
R_g	gas constant
R_0	initial bubble radius
S	bubble surface area
T	temperature in liquid
t_a	dimensionless time defined in equation (41a)
T_b	bubble temperature
T_g	gas phase temperature profile
u_r'	radial gas phase velocity
u_r	radial liquid velocity
V	bubble volume
v_r	gas-liquid interface velocity
X_g	gas molar fraction in bubble
X_v	vapor molar fraction in bubble
Z, B	parameters defined in equations (33) and (37) respectively

Greek characters

α	thermal diffusivity of liquid
β	bubble growth parameter (see equation (13))
ΔH	phase change enthalpy
η	r/R
λ	D/α
ρ_b, c_{pl}	density and specific heat capacity of the liquid
ρ_v	vapor concentration in the bubble
τ	normalized temperature

Subscript

“f”	external (far field) liquid conditions
Overbar	normalized variables

Appendix A. Derivation of governing bubble growth equations (4) and (5)

Here a detailed derivation of the governing equations for gas bubble growth in a volatile liquid will be presented. Only the mass/heat transfer dominated regime will be considered (i.e. far from the very incipient stages of growth). In this case the Rayleigh-Plesset equation degenerates to

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$P_b = P_g + P_v$. Any variation of pressure is transferred in the speed of sound in the bubble (time scale of microseconds) so pressure is considered to be uniform in the bubble. The typical solubility of gases in water is of the order of 10^{-4} in terms of molar fraction so the following assumptions holds: (i) The density of liquid does not depend on dissolved gas concentration (ii) The vapor pressure of liquid does not depend on dissolved gas concentration (according to Raoult law for the extremely low gas content in the liquid). Another great simplification arises from the fact that the typical ratio of gas to liquid density is of the order of 10^{-3} so a zero value can be safely assumed (i.e. $\rho_b/\rho_l = 0$). Based on the above assumptions the following equations are derived:

total mass jump balance on the gas liquid interface:

$$\rho_l(u_r - v_r) = \rho_b(u_r' - v_r) \quad (A1)$$

where u_r is the liquid velocity, prime refers to gas velocity and $v_r = dR/dt$ is the interface velocity. In the particular case of ρ_b/ρ_l negligibly small the relation degenerates to $u_r = v_r = dR/dt$. The continuity equation in the liquid domain (considering the constant liquid density) takes the form $\frac{1}{r^2} \frac{\partial^2 u_r}{\partial r^2} = 0$ which after integration and application of the boundary condition at the interface leads to

$$u_r = \frac{R^2}{r^2} \frac{dR}{dt} \quad (A2)$$

Using this velocity in the transient convection-diffusion equations for mass and heat transfer in liquid phase the following equations arises:

$$\frac{\partial c}{\partial t} + \frac{R^2 \dot{R}}{r^2} \frac{\partial c}{\partial r} = \frac{D}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial c}{\partial r} \right) \quad (A3)$$

$$\frac{\partial T}{\partial t} + \frac{R^2 \dot{R}}{r^2} \frac{\partial T}{\partial r} = \frac{\alpha}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) \quad (A4)$$

Let us consider the gas phase now. The incompressibility condition does not hold here since the bubble density can be vary due to pressure or temperature variation. The continuity equation in this case is

$$\frac{\partial \rho_b}{\partial t} + \frac{1}{r^2} \frac{\partial \rho_b r^2 u_r'}{\partial r} = 0 \quad (A5)$$

The above equation is integrated for the radial velocity in gas using as boundary condition $u_r' = 0$ at $r = 0$ (arising from spherical symmetry).

$$u_r' = -\frac{1}{r^2 \rho_b} \int_0^r x^2 \left(\frac{\partial \rho_b}{\partial t} \right) dx \quad (A6)$$

This velocity term is zero in case of growth under constant pressure and temperature. In case of varying pressure or varying (uniform) temperature, the density time derivative is uniform through the bubble and the velocity takes a linear profile:

$$u_r' = -\frac{r}{3\rho_b} \frac{d\rho_b}{dt} \quad (A7)$$

This is the velocity field induced in an inert bubble (no volatile liquid, no dissolved gas) due e.g. to its expansion as the ambient pressure decreases.

The conservative form of the heat transfer equation in the bubble is used to account for gas density spatial nonuniformities:

$$\frac{\partial \rho_b T_g}{\partial t} + \frac{1}{r^2} \frac{\partial r^2 u_r' \rho_b T_g}{\partial r} = \frac{k_b}{c_{pb} r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T_g}{\partial r} \right) \quad (A8)$$

where T_g , k_b , c_{pb} are the temperature, thermal conductivity and specific heat capacity in the bubble respectively. Conductivity and heat capacity are assumed to take constant values since their temperature dependence is smaller than the one of the density. However is quite straightforward to consider them temperature and compositional dependent.

Regarding the mass transfer problem in the bubble, it is necessary to work with molar fractions instead of concentrations to avoid the complications arising from the dependence of concentration on temperature. If X_g and X_v are the molar fraction of the gas and vapor, their distribution are dictated from the equations (again the conservative form is employed):

$$\frac{\partial \rho_b X_g}{\partial t} + \frac{1}{r^2} \frac{\partial r^2 u_r' \rho_b X_g}{\partial r} = \rho_b \frac{D_{gv}}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial X_g}{\partial r} \right) \quad (A9)$$

$$\frac{\partial \rho_b X_v}{\partial t} + \frac{1}{r^2} \frac{\partial r^2 u_r' \rho_b X_v}{\partial r} = \rho_b \frac{D_{gv}}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial X_v}{\partial r} \right) \quad (A10)$$

where D_{gv} is the binary vapor-gas diffusivity (in gas phase).

It can be shown (simply by adding (A9) and (A10)) that instead of solving both equations, we can solve only equation (A9) and find X_v from the relation

$$X_v = 1 - X_g \quad (A11)$$

The next step is to define the boundary conditions. The first is the temperature continuity at the interface:

$$T = T_b \quad \text{at} \quad r = R \quad (A12)$$

The second and the third conditions are imposed by thermodynamic:

- Equilibrium between gas and liquid phase concentration of the gas (Henry law)

$$c = HP_b X_g \quad \text{at} \quad r = R \quad (\text{A13})$$

- the vapor concentration at the interface should be the vapor pressure of the liquid

$$P_b X_v = P_v(T) \quad \text{at} \quad r = R \quad (\text{A14})$$

In case of water it is more convenient (and equivalent) to use an empirical relation for $P_v(T)$ (i.e. modified Antoine law) instead of the Clausius-Clapeyron equation. The next three boundary conditions are the jump balances at the interface.

They have the following form [14].

$$[flux]_l - Bv_r = [flux]_b - Bv_r \quad \text{at} \quad r = R \quad (\text{A15})$$

where flux terms in the liquid and gas phase have diffusional (conductive) and convective contributions and B is the variable under consideration (i.e. enthalpy, gas concentration, vapor concentration).

An order of magnitude analysis revealed that here no elimination of terms multiplied by ρ_b is possible as in equation (A1).

Application to the temperature field leads to

$$-k_b \frac{\partial T_g}{\partial r} + \rho_b c_{pb} \left(u_r - \frac{dR}{dt} \right) T_g + k \frac{\partial T}{\partial r} = K \Delta H \quad \text{at} \quad r = R \quad (\text{A16})$$

where K is the evaporation rate (moles/m²).

Application to gas concentration leads to

$$-D_{gv} \rho_b \frac{\partial X_g}{\partial r} + \rho_b \left(u_r - \frac{dR}{dt} \right) X_g = -D \frac{\partial c}{\partial r} \quad \text{at} \quad r = R \quad (\text{A17})$$

Finally application to the vapor concentration leads to (the liquid is assumed to be a single phase since the dissolved gas concentration is extremely small):

$$-D_{gv} \rho_b \frac{\partial X_v}{\partial r} + \rho_b \left(u_r - \frac{dR}{dt} \right) X_v = -K \quad \text{at} \quad r = R \quad (\text{A18})$$

Additional boundary conditions needed at $r = 0$:

$$\frac{\partial X_v}{\partial r} = \frac{\partial X_g}{\partial r} = \frac{\partial T_g}{\partial r} = 0 \quad (\text{A19})$$

The auxiliary relation $\rho_b = P_b / (R_g T_g)$ is also needed to close the problem. In addition $P_v = X_v P_b$, $P_g = X_g P_b$, $\rho_v = X_v \rho_b$, $\rho_g = X_g \rho_b$. The far field boundary conditions in the liquid phase are also needed

$$T = T_{ex}, C = C_{ex} \quad \text{at} \quad r \rightarrow \infty \quad (\text{A20})$$

The bubble growth model consists from partial differential equations (A3), (A4), (A8), (A9). The second and third of the conditions in equation (A19) (the first is dependent to the second) and the two far field conditions in equation (A20) defines uniquely the problem at $r = 0$ and $r \rightarrow \infty$. Only four conditions are needed to be defined at the interface in order to match the heat and mass balances in the two phases. However there are six independent to each other boundary conditions at the interface (equations (A12), (A13), (A14), (A16), (A17), (A18)). The two degrees of freedom arising from the additional constraints are employed to find the problem unknowns dR/dt (bubble growth rate) and K (evaporation rate).

The system of the derived equations is an innovative one even for the case of simple degassing since all the relevant studies starts with the assumption of uniform temperature and concentration in the bubble. Even in the detailed analysis of Scriven [12], it is stated that ‘‘Under all but the most extreme conditions compressibility effects, vapor inertia and viscosity, and pressure, temperature, and concentration gradients within the vapor can be disregarded’’ The most interesting aspect of the derivation here is the existence of the gas phase velocity u_r which is typically considered as zero [25].

The next step is to check the time scales of the transfer processes in the system (heat transfer in the gas, heat transfer in the liquid, mass transfer in the gas, mass transfer in the liquid). The four time scales are inversely proportional to the corresponding diffusivities. For a typical gas-liquid system (i.e. air-water) the heat and mass transfer diffusivities are scaled as 10^{-5} (gas), 10^{-7} (liquid) for heat, 10^{-5} (gas), 10^{-9} (liquid) for mass. It is clear that the dynamics in the liquid phase is at least two orders magnitudes slower than in the gas phase (a similar scaling holds for any gas liquid system). In the particular problem treated here any disturbance in the conditions is coming from the liquid and it is transferred to the bubble through the slow liquid phase dynamics. The dynamics in the bubble is much faster so it has all the time to response to the slow liquid dynamics. Since the bubble covers a restricted spatial domain the above assertion implies that the conduction/diffusion has all the time to create a uniform temperature/concentration profile. This is similar to the well known limit of large Biot number for transient heat transfer. It is worth noted that the situation is quite different in the case of droplet evaporation. There the slow thermal dynamics is in the restricted domain so there is always a droplet temperature profile. The fast dynamics is in the unrestricted domain where a pseudosteady heat transfer condition may be established but again a temperature profile exists connecting the droplet surface temperature to the far field gas temperature.

In the present situation the governing equation in the fast dynamics domain can be integrated through the whole domain. Then assuming uniformity of the dependent variable and employing the corresponding boundary conditions, an ordinary differential equation for the average temperature or concentration arises. So taking the integral of equation (A8) from $r = 0$ to R (after multiplying by r^2 to account for spherical symmetry), applying the Reynolds transport theorem and substituting the boundary condition (A16) leads to the following evolution equation for the bubble temperature (details appears in Appendix B)

$$c_{pb} \frac{1}{3R^2} \frac{dR^3 T_g \rho_b}{dt} = k \left(\frac{\partial T}{\partial r} \right)_{r=R} - K \Delta H \quad (\text{A21})$$

The evaporation rate K can be found by a global vapor mass balance as

$$K = \frac{1}{3R^2} \frac{dR^3 \rho_b X_v}{dt} \quad (\text{A22})$$

It is noted that the first term in equation (A21) stands for the thermal inertia of the bubble. However any change of the bubble temperature requires a change of a comparable (to the bubble volume) volume of liquid around the bubble (because of the temperature continuity at the interface). The thermal inertia of this volume of liquid is huge comparable to that of the bubble (the assumption $\rho_b/\rho_l \rightarrow 0$ has been already used). Omitting the first term in equation (A21) and substituting (A22) an equation equivalent to (5) (main section) is taken.

Applying a similar procedure to equation (A9) using the boundary condition (A17) leads to:

$$\frac{1}{3} \frac{dR^3 \rho_b X_g}{dt} = R^2 D \left(\frac{\partial c}{\partial r} \right)_{r=R} \quad (\text{A23})$$

which is actually the equation (4) of the main section.

In order to show the consistency and validity of the model we will show how equation (A23) can be derived without integration of (A9) but directly from the interfacial jump balances (A18) and (A19). Adding (A18) and (A19) and employing (A11) leads to:

$$\rho_b \left(u_r - \frac{dR}{dt} \right) = -D \frac{\partial c}{\partial r} - K \quad \text{at } r = R \quad (\text{A24})$$

In case of uniform temperature the gas phase velocity at the interface can be found from equation (A7) as

$$u_r = -\frac{R}{3\rho_b} \frac{d\rho_b}{dt} \quad (\text{A25})$$

Substituting (A25) and (A22) in (A24) leads to:

$$\frac{R}{3} \frac{d\rho_b}{dt} + \rho_b \frac{dR}{dt} = D \frac{\partial c}{\partial r} + \frac{1}{3R^2} \frac{dR^3 \rho_b X_v}{dt} \quad (\text{A26})$$

Substituting $X_v = 1 - X_g$ in (A26) equation (A23) results.

equation (A26) is a physically meaningful decomposition of (A25). The first term is the contribution to the bubble growth of the expansion/contraction due to ambient pressure/temperature variation, the third term is the contribution of the degassing and the last term the contribution of the evaporation.

Appendix B. Integration of conservation equations for the bubble interior

In this section the details of integration of equation (A8) are presented. Similar procedure has been used for equation (A9) and (A10).

At first the operator $\int_0^R r^2 \{ \dots \} dt$ is applied to all terms of equation (A8) and after performing the integrations leads to:

$$\int_0^R r^2 \frac{\partial \rho_b T_g}{\partial t} dr + [r^2 u_r \rho_b T_g]_{r=0}^{r=R} = \frac{k_b}{c_{pb}} \left[r^2 \frac{\partial T_g}{\partial r} \right]_{r=0}^{r=R} \quad (\text{B1})$$

The first term can be rewritten as $\frac{1}{3} \int_0^R \frac{\partial \rho_b T_g}{\partial t} dr^3$. The Reynolds Transport theorem can be applied to this term to give:

$$\frac{1}{3} \int_0^R \frac{\partial \rho_b T_g}{\partial t} dr^3 = \frac{1}{3} \frac{d}{dt} \int_0^R \rho_b T_g dr^3 - \rho_b T_g \frac{1}{3} \frac{dR^3}{dt} \quad (\text{B2})$$

Substitution of (B2) in (B1) using (A19) leads to:

$$\frac{1}{3} \frac{d}{dt} \int_0^R \rho_b T_g dr^3 - \rho_b T_g \frac{1}{3} \frac{dR^3}{dt} + R^2 u_r(R) \rho_b T_g = \frac{k_b}{c_{pb}} R^2 \left(\frac{\partial T_g}{\partial r} \right)_{r=R} \quad (\text{B3})$$

Substitution of the partial derivative of the gas temperature at the interface using (A16) leads to

$$\frac{1}{3} \frac{d}{dt} \int_0^R \rho_b T_g dr^3 = R^2 k \left(\frac{\partial T}{\partial r} \right)_{r=R} - R^2 K \Delta H \quad (\text{B4})$$

This trivially leads to (A21) in case of uniform temperature in the bubble.

Appendix C. Supplementary data

Supplementary data to this article can be found online at <https://doi.org/10.1016/j.ijthermalsci.2019.106114>.

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